A Bandwidth-Power Efficient Modulation Scheme Based on Quaternary Quasi-Orthogonal Sequences

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***Abstract*—A novel modulation scheme suitable for noncoherent demodulation based on quaternary quasi-orthogonal sequences is proposed. Compared to orthogonal modulation, the controlled quasi-orthogonality between the sequences allow significantly increased bandwidth efficiency with little or no degradation in power efficiency. A hardware efficient demodulator structure using fast Walsh transforms is also presented.**

***Index Terms*—Modulation, noncoherent demodulation, quasi-orthogonal sequences (QOSs).**

I. Introduction

ORTHOGONAL modulation (OM) schemes such as M-ary frequency shift keying (MFSK) [2] and M-ary Walsh modulation [3] combined with noncoherent demodulation are frequently used in communication systems where reliable phase recovery is not practically feasible or undesirable. It is well known that M-ary OM asymptotically achieves channel capacity under the AWGN channel as $M \to \infty$ with both coherent and noncoherent demodulation [2]. However, OM requires bandwidth which increases linearly with M for a given data rate, rendering them unsuitable for communication systems requiring bandwidth efficiency.

Modifying conventional OM schemes in order to increase the bandwidth efficiency by sacrificing the orthogonality among the signals have previously been addressed. Two examples include MFSK with nonorthogonal frequency spacing [2] and multitone MFSK (mMFSK) [4], both succeeding in increasing the bandwidth efficiency but only at the cost of significant loss in power efficiency.

In this letter, we propose a novel modulation scheme suitable for noncoherent demodulation based on quaternary quasi-orthogonal sequences (QOSs) [1] which will be referred to as quasi-orthogonal modulation (QOM). Compared to OM, we are able to achieve drastically increased bandwidth efficiency with little or no loss in power efficiency. A hardware efficient demodulator structure using the fast Walsh transforms (FWT) [5] is also presented.

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II. Quaternary QOSs

Let $\mathbf{x} = (x(0), x(1), \ldots, x(N-1))$ be a sequence of length N, where each $x(n)$ is referred to as a *chip*. The sequence $\mathbf{x}$ is called a *binary* sequence if $x(n) \in GF(2)$ and a *quaternary* sequence if $x(n) \in \mathbb{Z}_4$, the ring of integers modulo 4. Let $\mathcal{W}_m = \{\mathbf{w}_i | i = 0, 1, \ldots, 2^m - 1\}$ for a given positive integer m, be the set of binary Walsh sequences of length $N = 2^m$. Then, let $2\mathcal{W}_m = \{2\mathbf{w}_i | i = 0, 1, \ldots, 2^m - 1\}$ be the set of quaternary sequences derived from $\mathcal{W}_m$ by multiplying each sequence in $\mathcal{W}_m$ by 2 over $\mathbb{Z}_4$.

We define the correlation between two quaternary sequences $\mathbf{x}$ and $\mathbf{y}$ as $R_{\mathbf{x},\mathbf{y}} \triangleq \sum_{n=0}^{N-1} j^{x(n)-y(n)}$ with $j = \sqrt{-1}$. The set $\mathcal{F} = \{\mathbf{f}_i | i = 0, 1, \ldots, M-1\}$ of M quaternary QOSs of length $N = 2^m$ is then defined as a set of quaternary sequences satisfying the following two properties[1], [2] [1]:

1) $\mathcal{F}$ contains $2\mathcal{W}_m$;
2) for any two distinct sequences $\mathbf{f}_i, \mathbf{f}_l \in \mathcal{F}$, we have $|R_{\mathbf{f}_i,\mathbf{f}_l}| \leq \sqrt{N}$.

It was shown in [1] that a set of quaternary QOSs may be constructed based on an appropriate permutation of *Family* $\mathcal{A}$ sequences [6] and their cyclic shifts. The resulting set of quaternary QOSs of length N has a family size of $M = N^2$ and can be partitioned into N nonoverlapping equal size subsets $\mathcal{F}_L = \mathbf{c}_L \oplus_4 2\mathcal{W}_m = \{\mathbf{f}_{LN+i} | \mathbf{f}_{LN+i} = \mathbf{c}_L \oplus_4 2\mathbf{w}_i, 2\mathbf{w}_i \in 2\mathcal{W}_m, i = 0, 1, \ldots, N-1\}, L = 0, 1, \ldots, N-1$. Here, $\mathbf{c}_L$ is the defining *masking sequence*[3] for $\mathcal{F}_L$ and $\oplus_4$ denotes elementwise addition in $\mathbb{Z}_4$. The masking sequences for $N = 4, 8, 16,$ and 32 are given in Table I. Clearly, any two distinct QOSs contained in the same subset are orthogonal to each other. On the other hand, the correlation between any two QOSs contained in different subsets can be shown to take on only the four values $\pm\omega\sqrt{N}$, $\pm j\omega\sqrt{N}$ where $\omega = 1$ for even m and $\omega = e^{j\pi/4}$ for odd m [1].

III. System Model

For QOM, $k = \log_2 M$ data bits are used to choose a sequence from the set of M quaternary QOSs of length $N = \sqrt{M}$. The selected sequence is then transmitted using QPSK modulation[4]. The receiver consists of a chip pulse matched filter

[1] A third property called the *window property* is also included in [1] which places an upper bound on the absolute value of the *partial* correlation between any $\mathbf{f}_i$ in $\mathcal{F}$ but not in $2\mathcal{W}_m$ and any $2\mathbf{w}_i$ in $2\mathcal{W}_m$.

[2] If we change condition (b) to $|R_{\mathbf{f}_i,\mathbf{f}_l}| < \sqrt{N}$, then $\mathcal{F}$ reduces to $2\mathcal{W}_m$ since for any quaternary sequence $\mathbf{x}$ not contained in $2\mathcal{W}_m$, there exists at least one sequence $2\mathbf{w}_i \in 2\mathcal{W}_m$ such that $|R_{\mathbf{x},2\mathbf{w}_i}| \geq \sqrt{N}$ [1]. Thus, condition (b) implies that the correlation between any two distinct sequences in $\mathcal{F}$ should be as small as possible while not reducing $\mathcal{F}$ to $2\mathcal{W}_m$.

[3] For details on deriving the masking sequences, refer to [1].

[4] QPSK modulation of chip $f(n)$ is assumed to be given as $j^{f(n)}$.