

SHELDON M. ROSS

STOCHASTIC PROCESSES

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AND MATHEMATICAL STATISTICS**



STOCHASTIC PROCESSES

S e c o n d E d i t i o n

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