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Lutz Plümer

Termination Proofs
for Logic Programs



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Preface

This volume presents a technique for the automatic generation of termination proofs for logic programs. Such proofs can easily be achieved as long as recursive calls operate on arguments which are proper subterms of those originally given. If a procedure operating on recursive data structures has local variables in its recursive literals, termination proofs are difficult. Simplification orderings, which are often used to prove termination of term rewriting systems, are not sufficient to handle these cases. We therefore introduce the notion of linear predicate inequalities. These compare the sizes of tuples of terms of literals occurring in the minimal Herbrand model of a program. Term sizes are measured by linear norms. A technique for the automatic derivation of valid linear inequalities is described. This technique is based on the concept of AND/OR dataflow graphs. An algorithm which uses linear inequalities in termination proofs is given. The assumption that all recursion is direct keeps our approach efficient. We show, however, how mutual recursion can be eliminated by static program transformation. Finally we discuss how the scope of our technique can be enlarged by the integration of unfolding techniques.

This book is a slightly revised version of my thesis [PLU89]. It is based on research carried out within the EUREKA project EU 56 PROTOS. Research has been made possible by a grant from the Minister für Wissenschaft und Forschung des Landes Nordrhein-Westfalen. Partners of the PROTOS project, which started in April 1987 for a duration of three years, have been ETH Zürich, Schweizerische Bankgesellschaft, Sandoz AG, Belgian Institute of Management, IBM Deutschland, Universität Dortmund and Universität Oldenburg.

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