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Preface

TLCA'93 is the first international conference on Typed Lambda Calculi and Applications. It has been organised by the Department of Philosophy of Utrecht University, with kind assistance of the Centre for Mathematics and Computer Science (CWI) in Amsterdam, both in The Netherlands. Thanks go to these organisations for providing a financial backup that enabled us to undertake the organisation.

The 51 papers submitted were mostly of very good quality. From these, 29 papers were selected for presentation at the conference. We are glad, and even a little proud, to be able to say that almost all leading researchers in the area of Typed Lambda Calculi contribute to these proceedings. We thank the program committee for their careful selection of the best among the submitted papers.

Given the current developments in the area of typed lambda calculi and their applications, we think and hope that this conference will mark the beginning of a tradition of conferences covering this field.

Utrecht, January 1993 — The Editors

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On Mints' Reduction for ccc-Calculus

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Abstract. In this paper, we present a divide-and-conquer lemma to infer the SN+CR (Strongly Normalization and Church-Rosser) property of a reduction system from that property of its subsystems. Then we apply the lemma to show the property of Mints' reduction for ccc-calculus with restricted η-expansion and restricted π-expansion. In the course of the proof, we obtain some relations of the two restricted expansions against traditional reductions. Among others, we get a simple characterization of the restricted η-expansion in terms of traditional β- and η-reductions, and a similar characterization for the restricted π-expansion.

1 Introduction

By ccc-calculus, we mean a deduction system for equations of typed λ-terms, essentially due to Lambek-Scott[9]. The types are generated from the type constant T and other atomic types by means of *implication* $(\varphi \supset \psi)$ and *product* $(\varphi \times \psi)$. We use $\varphi, \psi, \sigma, \tau, \ldots$ as meta-variables for types.

The terms of ccc-calculus are generated from the constant $*^{\mathrm{T}}$ of type T and denumerable variables $x^{\varphi}, y^{\varphi}, z^{\varphi}, \ldots$ of each type φ by means of *application* $(u^{\varphi \supset \psi} v^{\varphi})^{\psi}$, *abstraction* $(\lambda x^{\varphi}. u^{\psi})^{\varphi \supset \psi}$, *left-projection* $(\mathrm{l} u^{\varphi \times \psi})^{\varphi}$, *right-projection* $(\mathrm{r} u^{\varphi \times \psi})^{\psi}$, and *pairing* $\langle u^{\varphi}, v^{\psi} \rangle^{\varphi \times \psi}$. We use $s^{\varphi}, t^{\varphi}, u^{\varphi}, v^{\varphi}, \ldots$ as meta-variables for terms of each type φ.

This system is to deduce equations of the form $u^{\varphi} = v^{\varphi}$ and it consists of the usual equational axioms and rules(i.e., reflexive, symmetric, and transitive laws and substitution rules) and the following proper axioms[2]:

- (β) $(\lambda x. u)v = u[x := v]$, where $[x := v]$ is a substitution;
- (l) $\mathrm{l}\,\langle u_{\mathrm{l}}, u_{\mathrm{r}} \rangle = u_{\mathrm{l}}$;
- (r) $\mathrm{r}\,\langle u_{\mathrm{l}}, u_{\mathrm{r}} \rangle = u_{\mathrm{r}}$;
- (η) $\lambda x. ux = u$ if variable x does not occur free in u;
- (π) $\langle \mathrm{l}u, \mathrm{r}u \rangle = u$;
- (T) $u^{\mathrm{T}} = *^{\mathrm{T}}$.

* Supported by JSPS Fellowship for Japanese Junior Scientists

[2] In this paper, the type forming operator $\supset$ is right associative. And the type-superscripts of terms are often omitted. The term forming operators l and r have higher precedences than the application operator. As usual, we denote α-congruence by $\equiv$. We refer to [1] as the standard text.

The proper axioms reflect the properties of cartesian closed category(CCC, for short), e.g. the axiom (T) reflects the property of the terminal object.

In relation to its decision problems, coherence problem and the like, it is desirable to have a reduction system to generate the calculus which is both Church-Rosser and (weakly or strongly) normalizable.

A naive idea to get such a system is to read each proper axiom (X) as a rewriting rule $\rightarrow_X$(to rewrite the left-hand side to the right-hand side where $X = \beta$, l, r, η, π) and $\rightarrow_{\mathrm{T}}$(to rewrite u^{T} to $*^{\mathrm{T}}$ when $u \not\equiv *^{\mathrm{T}}$). But, unfortunately, the system so obtained is not Church-Rosser[9], although it is strongly normalizable.

So, Mints[10] introduced a new reduction system by replacing the rules $\rightarrow_\eta$ and $\rightarrow_\pi$ with a certain restriction of η-expansion(denoted by $\rightarrow_{\eta^*}$) and that of π-expansion(denoted by $\rightarrow_{\pi^*}$) , respectively. Thus his reduction system consists of the basic reduction $\rightarrow_{\mathrm{B}}:=\rightarrow_\beta \cup \rightarrow_{\mathrm{l}} \cup \rightarrow_{\mathrm{r}}$, the restricted expansion $\rightarrow_{\mathrm{E}}:=\rightarrow_{\eta^*} \cup \rightarrow_{\pi^*}$, and the terminal reduction $\rightarrow_{\mathrm{T}}$. Then it is proved by Čubrić[3] that the reduction $\rightarrow_{\mathrm{BET}}:=\rightarrow_{\mathrm{B}} \cup \rightarrow_{\mathrm{E}} \cup \rightarrow_{\mathrm{T}}$ is weakly normalizable and Church-Rosser. We call the reduction $\rightarrow_{\mathrm{BET}}$ Mints' reduction.

In literature, the restricted η-expansion $\rightarrow_{\eta^*}$ also arose from the study of program transformation. Hagiya[6] introduced the same notion $\rightarrow_{\eta^*}$ independently in order to study ω-order unification modulo $\beta\eta$-equality[7] in simply-typed λ-calculus. A weaker version of $\rightarrow_{\eta^*}$ also appeared in Prawitz[11] in connection with proof theory.

The feature of $\rightarrow_{\eta^*}$, in addition to the strong normalization property, is that a term in $\rightarrow_{\eta^*}$-normal form explicitly reflects the structure of its type. For example, if a term $t^{\varphi_1 \supset \varphi_2 \supset \cdots \supset \varphi_n \supset \tau}$ is in $\rightarrow_{\eta^*}$-normal form then $t \equiv \lambda x_1 x_2 \cdots x_n . s$. The $\rightarrow_{\mathrm{BE}}$-normal form is known as expanded normal form[11] in proof theory.

Thus, the reductions $\rightarrow_{\eta^*}$ and $\rightarrow_{\pi^*}$ are useful reductions in category theory, proof theory, and computer science. Nevertheless these reductions have not been fully investigated, because of their context-sensitiveness and non-substitutivity.

The main theorem of this paper is:

- Mints' reduction $\rightarrow_{\mathrm{BET}}$ satisfies SN+CR property.

Based on the result, we also show that

- The reduction $\rightarrow_{\eta^*}$ is exactly an η-expansion which is not β-expansion.
- The reduction $\rightarrow_{\pi^*}$ is exactly a π-expansion which is not a finite series of $(\rightarrow_{\mathrm{l}} \cup \rightarrow_{\mathrm{r}})$-expansion.

Mints' reduction $\rightarrow_{\mathrm{BET}}$ inherits the above mentioned annoying properties of the reduction $\rightarrow_{\mathrm{E}}:=\rightarrow_{\eta^*} \cup \rightarrow_{\pi^*}$. In proving that $\rightarrow_{\mathrm{BET}}$ satisfies SN+CR property, to overcome the annoyance, we will use the divide-and-conquer technique: We separate the annoying part $\rightarrow_{\mathrm{E}}$ from the rest $\rightarrow_{\mathrm{BT}}:=\rightarrow_{\mathrm{B}} \cup \rightarrow_{\mathrm{T}}$ in the reduction $\rightarrow_{\mathrm{BET}}$, and apply the following lemma with $\rightarrow_R=\rightarrow_{\mathrm{E}}$ and $\rightarrow_S=\rightarrow_{\mathrm{BT}}$:

Lemma. If two binary relations $\rightarrow_R$ and $\rightarrow_S$ on a set $U(\neq \emptyset)$ have SN+CR property, then so does $\rightarrow_{SR} := \rightarrow_S \cup \rightarrow_R$, provided that we have

$$\forall u, v \in U \left(u \rightarrow_S v \implies u^R \xrightarrow{+}_S v^R \right) ,$$

where u^R and v^R are the $\rightarrow_R$-normal forms of u and v respectively, and $\xrightarrow{+}_S$ is the transitive closure of $\rightarrow_S$.

Thus our proof of the SN+CR property of $\rightarrow_{\mathrm{BET}}$ amounts to verifying the same property of $\rightarrow_{\mathrm{E}}$ and $\rightarrow_{\mathrm{BT}}$ separately and verifying the following:

Claim. For any terms u and v, if $u \rightarrow_{\mathrm{BT}} v$ then $u^{\mathrm{E}} \xrightarrow{+}_{\mathrm{BT}} v^{\mathrm{E}}$.

The SN+CR property of $\rightarrow_{\mathrm{BET}}$ has been already proved by Jay[8] and independently by the author, by means of a modified version of Girard's reducibility method. Comparing the divide-and-conquer method with the reducibility method, the advantages of the former are summarized as follows:

- The claim above clarifies the relation of $\rightarrow_{\mathrm{E}}$-normalization against the reduction $\rightarrow_{\mathrm{BT}}$.
- The method is less tedious in that we need not prove the weak Church-Rosser property of $\rightarrow_{\mathrm{BET}}$.

Besides, of independent interest might be the lemma above.

The basic idea of the lemma is found implicitly in Hagiya's proof [5] of the strong normalization property of $\rightarrow_\beta \cup \rightarrow_{\eta^*}$ for simply-typed λ-calculus.

The outline of this paper is as follows: In Sect. 2, the precise definition of Mints' reduction is given. In Sect. 3, the lemma is proved. In Sect. 4, $\rightarrow_{\mathrm{E}}$ is proved to satisfy SN+CR property. In Sect. 5, at first the $\rightarrow_{\mathrm{E}}$-normal form of an arbitrary term t^φ is described by induction on the structure of t^φ and φ. Then based on the description, we verify above mentioned claim. It then yields the property of Mints' reduction $\rightarrow_{\mathrm{BET}}$ as above, since $\rightarrow_{\mathrm{BT}}$ is known to satisfy the property by the standard argument. As a corollary to the main theorem, we give simple characterizations of $\rightarrow_{\eta^*}$ and $\rightarrow_{\pi^*}$. In Sect. 6, we discuss another proof method of our main theorem and relating topics.

2 Mints' Reduction

Definition 1. The subterm occurrence u in a term $C[uv]$ is said to be *functional*, while the occurrence u in $C[\mathrm{l}u]$ or in $C[\mathrm{r}u]$ is *projective*($C[\,]$ stands for an arbitrary one-hole context).

We define binary relations $\rightarrow_{\eta^*}$ and $\rightarrow_{\pi^*}$ on terms.

Definition 2 (η^*-reduction). $C[u] \rightarrow_{\eta^*} C[\lambda z.\,uz] \overset{\text{def}}{\Longleftrightarrow}$

(0) z does not occur free in u ,
(1) The occurrence of u in $C[u]$ is non-functional, and
(2) u is not an abstraction.

In this case the occurrence of u in $C[u]$ is called a *η^*-redex*.